BICYCLE SHOP
SEVENTH GRADE LESSON GUIDE

LESSON OVERVIEW:

The Bicycle Shop task asks students to identify the constant of proportionality and identify graphically, in a table or algebraically the solution to a system of linear relationships.

Note: Expand -- what is the purpose of the task – i.e. what mathematical ideas will students grapple with via engaging in the task?

COMMON CORE STATE STANDARDS:

- **7.RP.2** Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and v erbal descriptions of proportional relationships.
    c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.
        1. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

NCTM ESSENTIAL UNDERSTANDINGS[1]:

1. Reasoning with ratios involves attending to and coordinating two quantities.
2. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
3. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
4. A number of mathematical connections link rations and fractions:
    a. Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
    b. Ratios are often used to make “part-part” comparisons, but fractions are not.
    c. Ratios and fractions can be thought of as overlapping sets.
    d. Ratios can often be meaningfully reinterpreted as fractions.
6. A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
8. A rate is a set of infinitely many equivalent ratios.

[1] NCTM (2010) Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.

DRIVING QUESTIONS:	SKILLS DEVELOPED:
• How can we decide if two quantities are in a proportional relationship using a context, table, graph and equation? • How can we find the solution to a system of linear equations using a table or a graph? • What does the solution to a system of linear equation mean in the context of a problem?	*Students will be able to:.* • *determine whether or not two quantities are in a proportional relationship using a variety of representations* • *identify the constant of proportionality in a table, context, graph and equation.* • *find the solution to a system of linear equations using a table or a graph.* • *Interpret the meaning of the solution to a system of linear equations within the context of a problem.*
MATERIALS: "Bicycle Shop" Task, Document Projector or Chart Paper	**GROUPING**: Students will begin their work individually, but will then work in pairs or triads.

SET-UP

Instructions to Students:

Using either a doc ument reader or overhead projector present the task to the class. Have one student read the question that follow the graph and tabular representations.

Ask the students: "What do you know?" "What is the question asking you?"

Inform the students that there are several ways to get the answers to the questions asked. First each individual must work alone for at least 5 minutes after which they will share their initial findings with their group. Then they will continue to work out a common solution.

Expectations that all students must adhere to: explain their thinking and reasoning, use correct mathematical language and symbols in their explanations or solutions, justify their solutions, make sense of other students' explanations; seek help from the teacher or students when they do not understand.

EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas

Private Think Time: Allow students to work individually for 3-5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using.

Small-Group Work: After 3-5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.
- asking students to explain in their own words, and build onto, what other students have said.

As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a "heads up" that you will be asking them to come to the front of the room. If a document reader is not available, give selected groups an OVH transparency or chart paper to write their solution on.

Possible Solution Paths

1. Making a table

Number of Days	Bike City	Bike Town
0	160	0
1	240	120
2	320	240
3	400	360
4	480	480
5	560	600
6	640	720

Possible Assessing and Advancing Questions

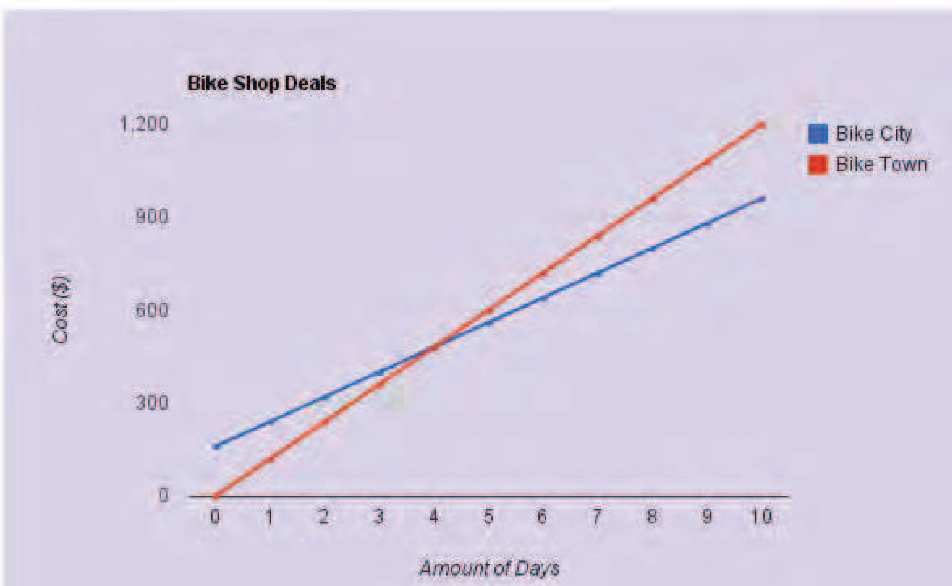
Assessing Questions

- What do the numbers represent in your table?
- How did you determine the numbers in your table?
- How does the table help you to solve the problem?

Advancing Questions

- Will there be another day at which two stores will charge the same amount? How do you know?
- Is there a proportional relationship between the number of days and the charge for either of the bike stores? How do you know?
- How do we see the daily rate for each of the bike shops in the table?

2. Drawing a graph



Assessing Questions

- What do the two lines represent in your graph?
- How does the graph help you to solve the problem?

Advancing Questions

- What do the points (0, 0) and (0, 160) mean the context of the problem?
- Is there a proportional relationship between the number of days and the charge for either of the bike stores? How do you know?
- What does the point (4,480) mean in the context of the problem?
- How do we see the daily rate for each of the bike shops in the graph?
- Will there be another day at which two stores will charge the same amount? How do you know?

3. Algebraic Solution (IF NO GROUPS ATTEMPT AN ALGEBRAIC SOLUTION, IT IS NOT NECESSARY TO PRESS FOR IT AT THIS TIME)

Bike City: $80x + 160 = y$
Bike Town: $120x = y$

$$80x + 160 = 120x$$
$$\underline{-80x \qquad - 80x}$$ *Step 1: Subtract 80x from both sides of the equal sign*

$$\frac{160}{40} = \frac{40x}{40}$$ *Step 2: Divide both sides by 40*

$4 = x$ *It will cost the same at Day 4.*

NOTE: The algebraic solution will not be discussed during the SDA phase since this was not a standard identified for this task. Explain to the students that you will just ask them to share, and explain, their equations.

Return to the algebraic solution for this task when you move to solving systems of linear equations.

Assessing Questions

- What do the two equations mean in the context of the problem? What does x represent? y? What do the 80, 160, and 120 represent?
- Why did you make the equations equal?
- What does the solution 4 = x mean in the context of the problem?

Advancing Questions

- Do either of these equations represent a proportional relationship? How do you know?
- How can we find the rate of change in the equations?
- Are either of the rates of change also a constant of proportionality? How do you know?

Possible Errors and Misconceptions	Possible Questions to Address Errors and Misconceptions

Graphing Errors:

- Inconsistent intervals
- plotting coordinates incorrectly

Mistaking Bike City as proportional since it has a constant rate of change.

Thinking that any rate of change is also a constant of proportionality

Incorrect equations

Assessing Questions

- What have you done so far?

Advancing Questions

- What elements do graphs need to have in place in order to be accurate?
- Why does it help to plot this data on the same graph?
- Is there another way you can show the relationship between the x and y values for Bike City? Bike Town?
- Can you describe some similarities and differences between the two graphs?
- What's the rate of change (constant of proportionality)? What is value of y when x equals 0?
- We've looked at graphs that are proportional. How is Bike City's graph different from these? (Have graphs ready that show proportionality)

SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding

General Considerations:

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

Possible Sequence of Solution Paths	Possible Questions and *Possible Student Responses*
1. Start with a solution using a table	**Explain your group's solution.** • *We started with one day for Bicycle City and found our how much it would cost for that day. Then we decided to continue adding up to 6 days. After that we did the same thing for Bike Town only stopping when Bike Town. We found that on day 4 they both had the same charge.* **Are either of these relationships proportional? If so, how can you tell that by looking at the table? At the context?** • *The charge for Bike Town is a proportional relationship because on day one it doesn't cost anything.* • *You can also tell that bike town is proportional because when you double the number of days you double the charge. That doesn't happen for Bike City.* • *Bike City isn't proportional because you start out with a $160 charge that gets added on.*
2. Have students share their graphical solution	**What does your graph represent?** • *We thought that it would be an easy solution and we would be able to see when the two bike companies have the same day and charge.* **How can you tell when the two bike shops charge the same charge by looking at the graph?** • *When the two lines cross that's when the two shops charge the same amount for the same number of days.* **How can you tell if either of the relationships is proportional by looking at the graph?** • *I can tell that Bike Town's charge is a proportional relationship because the graph starts at (0,0).* **From the graph can you tell if the two companies will ever have the same day and charge again?** • *The lines won't ever meet again. The lines are going at different slants because Bike Town charges more for each day than Bike City.*

3. Have students share their equations **NOTE:** For the purpose of this lesson, focus ONLY on the equations themselves during the SDA phase, not the algebraic solution of systems of linear equations.	**Explain how you arrived at the two equations.** • *Since Bike City starts with a $160 charge and then adds $80 for each day, I came up with the equation 80x + 160 = y. x is the number of days it takes to build the bike. Bike Town just charges $120 per day so their equation is 120x = y.* **How can we tell which relationship is proportional by looking at the equations?** • *We can tell that Bike Town is a proportional relationship because there's nothing added. The charge will always be 120 times the number of days.* **So what is the constant of proportionality for Bike Town?** • *The constant of proportionality is 120. The charge will always be 120 times the number of days.* **Bike City charges $80 per day. Why isn't that also a constant of proportionality?** • *You can't just multiply the number of days by 80 to find Bike City's charge. You also have to add $120. A constant of proportionality is always a multiple.*
4. Look across the different representations	**We have seen three representations – tables, graphs, and equations. How do they each help us to determine if a relationship is proportional?** • *We can see in all of them that Bike Town charges $0 for 0 days. We can see that in the first row of the table. We see that on the graph because the line goes through (0,0). We see that in the equation because when you multiply a number by zero you end up with zero, and you're not adding anything else.* • *We can also see that the charge for Bike Town is always $120 times the number of days. In the table you can multiply to check. In the equation it's 120x. It's a little harder in the graph, but you see the line always goes up the same amount. It doesn't curve.* • *You don't see any of these for Bike City.*

CLOSURE
Quickwrite: How can you tell if a relationship is proportional by looking at the context, table, graph and equation?
Possible Assessment: • .Look at different graphs and identify which are proportional and which are not with explanations.
Homework: • .Similar problem with different numbers

Bicycle Shop

Two bicycle shops build custom-made bicycles. Bicycle City charges $160 plus $80 for each day that it takes to build the bicycle. Bike Town charges $120 for each day that it takes to build the bicycle.

For what number of days will the charge be the same at each shop?